UNIVERSITÉ DE MONTRÉAL

FUNCTIONAL FORM AND STOCHASTIC SPECIFICATION: CURRENT APPLICATIONS TO AGGREGATE TIME-SERIES AND CROSS SECTIONAL TRAVEL DEMAND MODELS

by

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ABSTRACT

This paper summarizes some of the current work on the functional forms of travel demand models. Typically, simple models, such as the linear regression models or the linear exponent LOGIT model, occur as nested special cases of more general forms which have less restrictive and often more interesting properties that the popular simple forms. The paper considers both types of models found in the travel demand literature: models of *level*, which explain the number of trips by each mode and *probabilistic market share* models, which explain a probability of choice or market share of a mode. In the section which discusses the former class, considerable attention is given to modelling the stochastic component of the models in order to obtain as much information as possible from the systematic components usually present in the error terms of a model; the impact of such modelling of the errors upon the optimal functional form of the "fixed" non stochastic part of the model is also considered. In the section which discusses the second class of models, a number of research orientations, notably concerning the development of a whole family of probabilistic or market share models, are described in some details

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RÉSUMÉ

Nous présentons dans cet article un résumé de travaux récents sur deux catégories de modèles de demande de transport: les modèles qui expliquent le *niveau* de demande des modes de transport et ceux qui expliquent la *probabilité de choix ou part de marché* des modes de transport. La perspective utilisée est celle de la formulation de modèles d'une forme assez générale pour comprendre comme cas particuliers les modèles usuels, tels le modèle linéaire de régression ou le LOGIT à fonctions linéaires. Nous montrons que les formes plus générales ont souvent des propriétés nouvelles intéressantes que ne possèdent pas les cas particuliers. Dans la section qui traite des modèles de niveaux, nous insistons sur la formulation adéquate d'un modèle des erreurs et sur l'impact de cette formulation sur la forme fonctionnelle optimale de la partie "fixe" ou non aléatoire du modèle considéré. Dans la seconde section, nous décrivons de manière assez précise des recherches en cours sur le développement d'une famille de nouveaux modèles probabilistes ou de parts de marché.

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1. INTRODUCTION

Consider a demand model consisting of a "fixed" part, which contains a vector $X=(X_1,\ldots,X_k)$ of variables and parameters ϕ,

$$y_t = f(X_{1_t},\ldots,X_{K_t};\phi) + u_t \qquad (1)$$

and of an error term u_t; this stochastic part contains a truly random element e_t and a systematic component :

$$u_t = s(Z_{t-\ell},\ldots,Z_{t-r};u_{t-\ell},\ldots,u_{t-r};e_t;\psi), \qquad (2)$$

where $Z=(Z_1,\ldots,Z_E)$ is a vector of explanatory variables and ψ denotes parameters.

In this two equation model, written with current and lagged observation subscripts $t=1,\ldots,N$ and $\ell=0,1,\ldots,r$, the first equation can be thought of as describing the causal structure, or the factors which "explain" y_t, and the second equation as a model of the error term u_t.

Our primary emphasis in this paper will be on the estimation of $f(\cdot)$, the functional form of the fixed part, for a given model of the stochastic part; our secondary emphasis will be on the importance of an adequate model of the error of a formulation $s(\cdot)$ which uses the systematic information contained in u_t in order to obtain efficient parameter estimates, notably of the functional form parameters of the fixed part.

The examples we will refer to pertain to aggregate data. We will first present models of *level*, where y_t denotes a number of trips, and introduce functional forms of interest; we will then examine *probabilistic or share* models which determine the probability of choice or market share of an alternative — in our case a travel mode. We will not survey the literature of models of level and of mode choice models but simply give a flavour of some recent developments and present orientations.

2. MODELS OF LEVEL

A. A time-series direct demand model with autocorrelated errors

The model we will use as a reference point is the DEMTEC direct demand time-series model. The fixed part of the model can be expressed as

$$d_{q_t} = f(P_t,T_t,C_t,Y_t,A_t,ETC_t),\ q=1,\ldots,M, \qquad (3)$$

where the level of travel demand in each market or mode q, during each period t, is assumed to depend on vectors of prices P_t, service levels T_t and comfort C_t of the relevant modes, income and socioeconomic characteristics T_t of the population, activity levels A_t and other variables ETC_t. This model belongs to the classic group of direct demand formulations proposed by Ruiter (1973). In the application of this model to Montréal, two markets have been principally focused on: the monthly transit demands by adults and by schoolchildren (Gaudry, 1978). The list of variables comprised in the first generation schoolchildren equation is shown in Table 1; this equation, assumed to be linear in parameters, will be our reference to analyse both problems of functional form of its fixed part and the specification of its stochastic part.

TABLE 1

TRANSIT DEMAND BY MONTREAL SCHOOLCHILDREN: LINEAR MODEL (DEC. 1956 - DEC. 1971)

(Experiment XSC 2-17-118-N)		(A) LSGAU estimator*		(B) ISUGAU estimator*		(C) IFIVER estimator**	
Variable	Code	Coefficient	t-stat.	Coefficient	t-stat.	Coefficient	t-stat.
P:Real transit fare	FSCD	-22,597,794.	-3.75	-22,413,647.	-3.70	-27,142.170.	-3.89
Number of cars	NC	-11.484	-2.30	-10.972	-2.16	-13.171	-2.45
Consum.price index	CPI	11,832.	0.36	8,648.1	0.26	48,677.	1.34
Y:Real average earn.	RAWEM	25,341.	1.39	25,700.	1.38	21,018.	1.09
C:Transit wait-time	TW	-166,420.	-0.79	-228,067.	-1.00	-61,635.***	-0.22
Transit in-veh.time	TT	-22,620.	-0.52	20,071.	0.46	99,115.***	1.26
Car in veh.time	T	136,326.	0.48	159,823.	0.55	-141,871.	-0.48
W:Temperature discomf.	TLK	1,067.	3.15	833.	2.55	500.	1.31
Rainfall discomf.	RFK	-51,312.	-1.19	-49,840.	-1.16	-72,509.	-1.60
Snowfall discomf.	SFK	-17,069.	-2.10	-18,877.	-2.32	-13,306.	-2.01
Cumul.snowf.discomf.	CSFMR	1,319.	1.02	1,560.	1.18	2,414.	1.71
A:School index	RSCMP	3.3129	8.06	3.2673	7.84	3.1171	7.47
Shopping index	SHONF	1.5854	2.86	1.5050	2.63	1.4179	2.47
Fair '67	EX7	.19312	4.75	.18823	4.50	.15361	3.23
Fair '68	EX8	.24043	1.55	.22175	1.40	.16957	1.05
Transit strikes	STR	-118,462.	-13.15	-118,173.	-13.13	-111,679.	-11.84
Trend	TEND	19,538.	2.72	19,666.	2.69	14,488.	1.96
ETC:Anticipatory sales	ANT	1,254,945.	5.78	1,220,156.	5.67	1,161,559.	5.56
Ant.sales lagged one	ANTL1	-430,819.	-1.78	-440,112.	-1.82	-400,948.	-1.70
Zonal double count.	ZSC	2,486,680.	3.70	2,792,077.	4.06	3,642,174.	4.51
Weekdays/month	WD	100,126.	2.92	109,166.	3.12	108,969.	3.06
Saturdays/month	SAT	37,424.	0.80	47,977.	1.02	38,056.	0.77
Sun.+ holidays/m.	SHD	32,644.	0.88	43,797.	1.16	34,032.	0.88
Fare card control	CHEAT2	-64,535.	-0.43	-72,554.	-0.48	-21,935.	-0.14
Line restructuration	STRUCT1	-257,829.	-0.98	-53,739.	-0.20	429,841.	1.00
Student card exp.	CARDEX	-107,921.	-0.68	-116,989.	-0.72	-127,225.	-0.80
Constant	C	-1,651,519.	-0.64	-2,634,091.	-1.01	-5,805,038.	-1.99
Serial correlation	ρ_1	.16	2.50	.18	2.80	.21	3.28
	ρ_3	-.31	-4.86	-.27	-4.19	-.27	-4.20
	ρ_4	-.06	-0.98	-.07	-1.04	-.09	-1.34
	ρ_{12}	.41	6.37	.42	6.85	.40	6.30
R-SQUARED ON TRANSFORMED y*		.84	---	.83	---	n.c.	---
R-SQUARED ON UNTRANSFORMED y		.93	---	.93	---	n.c.	---
DURBIN-WATSON STATISTIC		1.91	---	1.92	---	n.c.	---

* 15 iterations. ** 40 iterations. *** Treated as endogeneous variable.

In a time-series analysis, one does not expect the error term u_t to be random. In this case, we write the multiple order autoregressive structure

$$u_t = \sum_{\ell=1}^{r} \rho_\ell \, u_{t-\ell} + e_t \qquad (4)$$

where the e_t's are independent and assumed to be identically distributed with zero mean and constant variance. Box-Jenkins (1970) techniques are used to identify the significant ρ_ℓ's: equation (4) is an autoregressive approximation of what may in effect be a mixed autoregressive-moving average (ARMA) process.

Taken together, a linear form of (3) and (4) define a nonlinear equation

$$y_t - \sum_{\ell=1}^{r} \rho_\ell y_{t-\ell} = \sum_{k=1}^{K} \beta_k \left(X_{k_t} - \sum_{\ell=1}^{r} \rho_\ell X_{k_{t-\ell}}\right) + e_t \qquad (5)$$

which we solve by the Cochrane-Orcutt (1949) iterative technique. A fully documented computer programme (Gaudry *et al.*, 1981) incorporates this approach which consists of combining the "profitable" approximate autoregressive model of errors and classical least-squares regression. Column (A) of Table 1 contains results obtained with this algorithm. Note that the autoregressive structure contains four parameters (ρ_1,ρ_3, ρ_4,ρ_{12}). Since 1975, the Montréal Transit Authority, the MUCTC, uses both adult and schoolchildren demand functions to explain and forecast ridership (and revenues) bi-annually. At present, the Toronto Transit Authority, the TTC, is implementing the DEMTEC approach with a four-market (golden age, adults, scholars, children) model.

We have also used the same approach to analyse urban gasoline sales and extended the system of equations (Gaudry, 1980) to include performance and supply equations, all defined over the urban area as a whole. The equations are estimated singly, grouped or simultaneously with a fully documented computer program (Gaudry and Liem, 1981) which extends to a set of equations the approach explicated above for the schoolchildren demand equation.

B. Introducing Box-Tukey and Box-Cox functional forms

We define the *direct* and *inverse* Box-Tukey transformations of a positive variable X_k respectively as

$$X_k^{(\lambda_k,\mu_k)} = \begin{cases} \dfrac{(X_k-\mu_k)^{\lambda_k}-1}{\lambda_k}, & 0 \le \mu_k < X_k, \ \lambda \neq 0 \\ \ell n\,(X_k-\mu_k), & 0 \le \mu_k < X_k, \ \lambda \to 0 \end{cases} \qquad (6)$$

and

$$X_k^{(\lambda_k,\mu_k)^{-1}} = \begin{cases} (\lambda_k X_k+1)^{\lambda_k^{-1}} + \mu_k, & \lambda \neq 0 \\ e^{X_k} + \mu_k, & \lambda \to 0 \end{cases} . \qquad (7)$$

The direct and inverse Box-Cox transformations are obtained by setting $\mu_k=0$. These definitions are consistent with the common use of the expression "Box-Cox transformation"; however, in the original paper by Box and Cox (1964), transformations with and without μ_k are discussed.

The fixed part of the model can be formulated as

$$y_t^{(\lambda_y,\mu_y)} = \beta_1 + \sum_{k=2}^{K} \beta_k X_{k_t}^{(\lambda_k,\mu_k)} \qquad (8)$$

The μ parameters in effect shift the natural coordinates of the variables and the λ parameters determine the form of the impact of each variable. Setting the μ parameters to zero, equation (8) contains as special cases a number of standard models: the linear ($\lambda_y=\lambda_1=\ldots=\lambda_K=1$), the multiplicative or log linear ($\lambda_y=\lambda_1=\ldots=\lambda_K=0$), and the exponential ($\lambda_y=0;\lambda_1=\ldots=\lambda_K=1$) obtained by inversing the transformation of the dependent variable. Generally speaking, there is of course no reason to expect parameter values to be 0 or 1: one can let the data reveal the optimal form and test whether that form is significantly different from standard cases or from one's strongly held theories or prejudices...

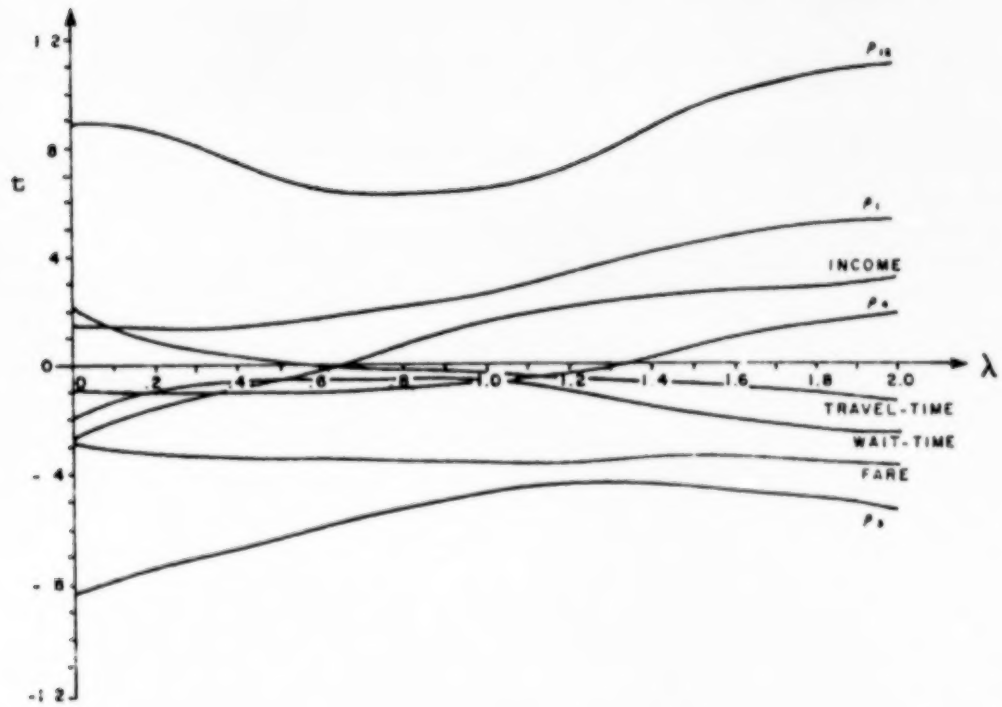
C. Application to the model under a constant error variance assumption

To show the impact of transformations, consider a simple case in which the shift parameters have been set to zero ($\mu_y=\mu_1=\ldots=\mu_K=0$) and the same power transformation used on both dependent and strictly positive independent variables ($\lambda_y=\lambda_1=\ldots=\lambda_K$) of the schoolchildren transit demand equation. In the stochastic part of the model, consider the structure defined by equation (4) and assume that its random error e_t has a constant variance σ^2. This assumption of constant variance, or homoscedasticity, has been made in all published Box-Cox analyses we have seen.

In this case, the optimum λ, estimated with a low degree of precision, was approximately .85 and, although significantly different from 0, was not significantly different from 1. The impact of the transformation on conditional t-statistics[@] associated with selected variables of the model and its four autocorrelation parameters are shown in Fig. 1. Corresponding elasticities of demand and autocorrelation parameter values are shown in Fig. 2. It is important to note that if the logarithmic model ($\lambda=0$) had been chosen, one would have concluded with some confidence (both t-statistics are around 2.00) that transit is an inferior good (the sign of the income variable is negative) and that children prefer long travel times (the sign of the transit travel time variable is positive); with the optimum λ or the linear model, the signs of these variables reverse themselves.

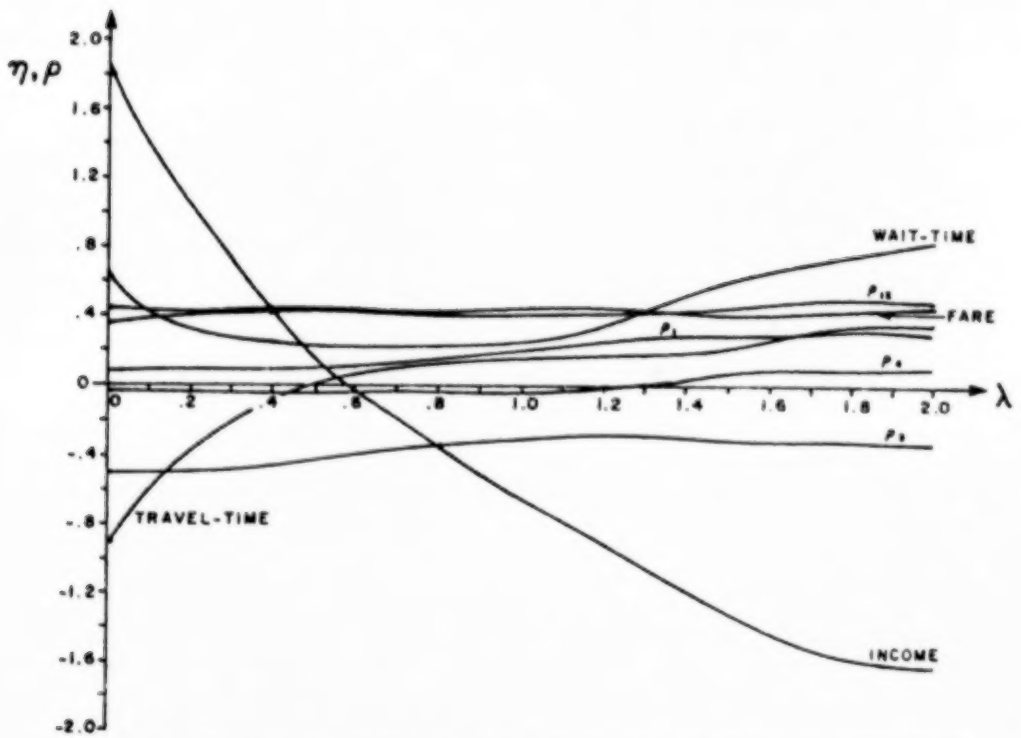
The paper from which these results are drawn (Gaudry and Wills,1978) contains results on the impact of a shift parameter μ_y used on the dependent variable and similar results on a transit demand equation for adults; it also contains results on an intercity total travel demand equation estimated from cross-sectional data with four Box-Cox transformations. It also contains analyses and results which shed doubts on the

[@] The t-statistics for the β_k are conditional on optimal values of the ρ_ℓ and λ; for the ρ_ℓ, they are conditional on the optimal values of the β_k and λ.



SCHOOLCHILDREN TRANSIT DEMAND EQUATIONS: CONDITIONAL t-STATISTICS FOR SELECTED VARIABLES AND THE AUTOREGRESSIVE PARAMETERS

FIGURE 1



SCHOOLCHILDREN TRANSIT DEMAND EQUATION: SELECTED ELASTICITIES AND VALUES OF AUTOREGRESSIVE COEFFICIENTS

FIGURE 2

appropriateness of the constant error variance assumption under which Box-Cox models are estimated. This question of heteroscedasticity we shall presently address before we discuss the importance of using a large number of Box-Cox transformations.

D. Model response with heteroscedasticity

The functional form estimates summarized above were obtained with a model of the stochastic part which takes into account multiple autocorrelation of the residual u_t but assumes that the variance of e_t in (4) is constant. The reasonableness of this assumption is open to question if one considers that error variance is a parameter of the system and consequently depends on the other parameters of the system. Clearly, Box-Cox transformations *simultaneously* affect the functional form of the regression model and the form of the density of the residual errors. A strategy to deal with this problem (Gaudry and Dagenais, 1979) consists of correcting for heteroscedasticity with a specific scheme at the same time as one improves the functional fit with the Box-Cox transformations. The model of errors can now be written

$$u_t = (\exp\{\delta_o + \sum_{m=1}^{M} \delta_m Z_{m_t}^{(\lambda_{zm})}\})^{\frac{1}{2}} v_t \ , \tag{9}$$

$$v_t = \sum_{\ell=1}^{r} \rho_\ell \, v_{t-\ell} + e_t \ . \tag{10}$$

In this model of errors, the Z_m are explanatory variables which may be the same as the X_k of the fixed part of the model. Expression (9) contains as special cases most forms of heteroscedasticity encountered in practice: for instance, the classical multivariate format is obtained by setting the $\lambda_{zm} = 0$ and the $\delta_m = 2$.

The impact of this correction for heteroscedasticity on the functional form estimates is shown in Table 2 which contains in column [3] the reference model used in sections A and C above. The results, however, are obtained with a more precise algorithm than those discussed in section C: the optimal λ, common to all strictly positive variables of the fixed part of the model, is 0.72 instead of 0.85 as above. This algorithm is incorporated in a completely documented computer program (Liem, 1979). Using one Z variable (TEND) improves the log likelihood value from 268 in column [3] to 277.28 in column [6] and using another variable (SHONF) instead increases it to 288.98 in column [7]; both specifications have a large impact on the optimal λ.

Table 2 also permits to note the considerable impact of failing to correct for autocorrelation, with or without simultaneously correcting for heteroscedasticity, as well as the impact of increasing the number of Box-Cox transformations from one to three: one (λ_y) on the dependent variable and two for groups ($\lambda_{x(1)}$ and $\lambda_{x(2)}$) of explanatory variables. In the latter case, we note that the change in log likelihood values between columns [8] and [9] does not justify the increased generality yielded by freeing two constraints. Additional work is in progress, using up to six distinct Box-Cox transformations, to understand whether the decreasing returns apparent here occur in other time-series models,

TRANSIT DEMAND BY MONTREAL SCHOOLCHILDREN: BOX-COX MODEL (Dec. 1956 - Dec. 1971)

TABLE 2 - Part A

Parameters \ Constraints	[1] $\lambda_y=\lambda_{x(1)}=\lambda_{x(2)}=1$ · no auto. · no hetero. (1) · no hetero. (2)		[2] $\lambda_y=\lambda_{x(1)}=\lambda_{x(2)}$ · no auto. · no hetero. (1) · no hetero. (2)		[3] $\lambda_y=\lambda_{x(1)}=\lambda_{x(2)}$ · no hetero. (1) · no hetero. (2)		[4] $\lambda_y=\lambda_{x(1)}=\lambda_{x(2)}$ · no auto. · no hetero. (2)		[5] $\lambda_y=\lambda_{x(1)}=\lambda_{x(2)}$ · no auto · eq. par. hetero (1) · no hetero. (2)	
Functional form										
λ_y	1.00	---	0.51	(2.67) (-2.56)*	0.72	(3.27) (-1.25)*	0.48	(2.18) (-2.34)*	1.05	(4.66) (0.24)*
$\lambda_{x(1)}$										
$\lambda_{x(2)}$										
Autocorrelation										
ρ_1	0.00	---	0.00	---	0.14	(1.78)	0.00	---	0.00	---
ρ_3					-0.34	(-3.60)				
ρ_4					-0.09	(-1.06)				
ρ_{12}					0.38	(4.52)				
Heteroscedasticity										
λ_{x1}	1.00**	---	1.00**	---	1.00**	---	2.14	(1.74) (0.92)*	0.00	---
λ_{x2}							1.00**	---	1.00**	---
δ_1 · TEMP (λ_{z1})	0.00	---	0.00	---	0.00	---	0.01	(0.36)	2.00	---
δ_2 · SHOMF (λ_{z2})							0.00	---	0.00	---
Selected Elasticities of Demand	η-E		η-E		η-E		η-E		η-E	
Real ticket fare FSCD ($\lambda_{x(1)}$)	-0.38	(-2.31)	-0.29	(-1.04)	-0.42	(-1.75)	-0.39	(-1.69)	-0.15	(-0.97)
Real weekly earnings EAWEN ($\lambda_{x(1)}$)	0.91	(1.49)	0.21	(0.27)	0.27	(0.43)	0.58	(0.68)	0.71	(0.88)
Transit vehicle-time TV ($\lambda_{x(1)}$)	-0.07	(-0.33)	-0.39	(-0.85)	-0.28	(-0.74)	-0.19	(-0.49)	-0.16	(-0.61)
School registration ESCHP	0.44	(11.84)	0.47	(4.42)	0.37	(3.99)	0.48	(4.08)	0.41	(3.90)
Population of children TEMP ($\lambda_{x(2)}$)	0.54	(2.18)	0.60	(1.73)	0.60	(1.86)	0.39	(1.14)	0.34	(0.87)
Real retail sales SHOMF ($\lambda_{x(2)}$)	0.14	(3.74)	0.13	(2.60)	0.11	(2.34)	0.14	(2.46)	0.15	(2.56)
Log likelihood $\mathcal{L}$	237.11		241.20		268.00		253.73		223.34	
Pseudo-(E)-R^2 ***	.907		.909		.934		.904		.885	
Pseudo-(L)-R^2	.907		.911		.935		.923		.890	
Max $\lvert Pr(y_t<0)\rvert$ ****	1.14×10^{-7}		0.00		7.11×10^{-15}		0.00		9.75×10^{-3}	

* Numbers in parentheses are asymptotic t-tests and an asterisk refers to the hypothesis that the parameter is equal to 1 instead of 0.
** Strictly speaking, when $\delta_{z(i)} = 0$, $\lambda_{x(i)}$ is undefined (i = 1,2).
*** See part 2 of this table.
**** See part 2 of this table.

TRANSIT DEMAND BY MONTREAL SCHOOLCHILDREN: BOX-COX MODEL (Dec. 1956 - Dec. 1971)

TABLE 2 - Part B

Parameters \ Constraints	[6] $\lambda_y=\lambda_{x(1)}=\lambda_{x(2)}$ = 0 (no hetero. (2))		[7] $\lambda_y=\lambda_{x(1)}=\lambda_{x(2)}$ = 0 (no hetero. (1))		[8] $\lambda_y=\lambda_{x(1)}=\lambda_{x(2)}$ = 0		[9] no constraints		[10] $\lambda_{x(1)}=\lambda_{x(2)}=1$	
Functional Form λ_y	1.00	(4.80) (0.02)*	1.22	(6.25) (1.11)*	0.80	(4.21) (-1.09)*	0.59	(2.90) (-2.04)*	0.68	(3.57) (-1.69)*
Functional Form $\lambda_{x(1)}$							1.52	(1.46) (0.50)*	1.00	---
Functional Form $\lambda_{x(2)}$							1.10	(1.89) (0.17)*	1.00	---
Autocorrelation ρ_1	0.25	(2.89)	0.19	(2.45)	0.57	(7.94)	0.50	(6.21)	0.60	(8.70)
Autocorrelation ρ_3	-0.19	(-1.74)	-0.11	(-1.23)	-0.36	(-4.04)	-0.39	(-3.98)	-0.37	(-4.15)
Autocorrelation ρ_4	-0.06	(-0.57)	-0.15	(-1.77)	0.13	(1.47)	-0.09	(1.02)	0.14	(1.60)
Autocorrelation ρ_{12}	0.47	(6.36)	0.46	(6.53)	0.38	(5.28)	0.36	(4.55)	0.38	(5.32)
Heteroscedasticity λ_{z1}	1.07	(1.04) (0.07)*	1.00	---	1.43	(1.91) (0.57)*	1.81	(1.90) (0.85)*	1.64	(1.94) (0.76)*
Heteroscedasticity λ_{z2}	1.00**	---	-10.41	(-8.53) (-9.35)*	4.71	(8.85) (6.97)*	4.69	(8.45) (6.65)*	4.88	(8.97) (7.13)*
Heteroscedasticity δ_1 · TEND (λ_{z1})	0.12	(0.48)	0.00	---	0.08	(0.64)	0.03	(0.49)	0.05	(0.56)
Heteroscedasticity δ_2 · SHONF (λ_{z2})	0.00	---	467.38	(2.73)	-0.05	(-1.56)	-0.05	(-1.52)	-0.04	(-1.51)
Selected Elasticities of Demand	η-E		η-E		η-E		η-E		η-E	
Real ticket fare FSCD ($\lambda_{x(1)}$)	-0.41	(-1.88)	-0.36	(-1.81)	-0.53	(-2.08)	-0.60	(-1.44)	-0.54	(-2.76)
Real weekly earnings RAWEM ($\lambda_{x(1)}$)	0.82	(0.97)	0.32	(0.55)	2.18	(4.72)	1.99	(0.47)	2.32	(2.97)
Transit wait-time TW ($\lambda_{x(1)}$)	-0.33	(-0.74)	0.14	(0.74)	-0.51	(-1.73)	-0.47	(-0.65)	-0.40	(-1.39)
School registration RSCMP	0.34	(4.05)	0.36	(4.69)	0.30	(4.85)	0.34	(4.86)	0.32	(5.02)
Population of children TEND ($\lambda_{x(2)}$)	0.57	(1.38)	0.36	(1.94)	0.96	(3.16)	0.93	(0.76)	0.90	(3.74)
Real retail sales SHONF ($\lambda_{x(2)}$)	0.09	(1.57)	0.05	(1.01)	0.19	(3.04)	0.18	(1.69)	0.18	(2.31)
Log likelihood $\mathcal{L}$	277.28		288.98		296.09		297.83		297.45	
Pseudo-(E)-R^2 ***	.932		.927		.924		.925		.925	
Pseudo-(L)-R^2	.942		.950		.954		.955		.954	
Max [Pr(y_t=0)] ****	6.28×10^{-6}		3.22×10^{-3}		5.85×10^{-6}		7.11×10^{-15}		1.59×10^{-9}	

* See part 1 of this table.

** See part 1 of this table.

*** The pseudo-(E)-R^2 is computed using the following formula: $1 - \sum_{t=1+r}^{N} (y_t - E(y_t))^2 / \sum_{t=1+r}^{N} (y_t - \bar{y})^2$. This measure does not necessarily lie between 0 and 1. The pseudo-(L)-R^2 is computed using the following formula: $1 - \exp(\frac{2}{N}(\mathcal{L}_0 - \mathcal{L}))$. In this expression, $\mathcal{L}_0$ is the value of the log-likelihood under the assumption that the model is linear with spherical residual errors and that all β's but the constant are equal to 0. This measure necessarily lies between 0 and 1 and increases monotonically with $\mathcal{L}$. In the classical linear regression model, both the pseudo-(E)-R^2 and the pseudo-(L)-R^2 coincide with the traditional R^2.

**** The probability that $y_t = 0$ is estimated for each observation and the maximum value is recorded here.

and whether they are related to the interaction between the fixed and stochastic parts of models. In this context, it is interesting to note in Table 2 that the real interaction between the model of errors and the estimated functional form of the fixed part, as measured by likelihood ratios, only partially carries over to the demand elasticities which are not very affected by the detailed specification of the model parts, once account is taken of the statistical significance of each variable.

All in all, we have excellent reasons to believe that the question of the influence of the efficient use of information contained in errors on functional form estimates is a useful topic of research, given the growing use of Box-Cox transformations in transportation (Kau and Sirmans (1976), Wang *et al.* (1980), Wills (1981)) and other areas where models of level are used.

Further details on the statistical model and algorithm used to obtain the results shown in Table 2 can be found in Dagenais *et al.* (1980).

3. PROBABILISTIC OR MARKET SHARE MODE CHOICE MODELS

We have shown, starting from the classical linear regression model, how flexible functional form could — with or without a detailed model of errors — improve the explanatory power of travel demand models. We now proceed to examine, starting from the popular linear exponent logit model, how various simple alternate forms can improve our understanding of mode choice behavior. A few general remarks can be helpful.

A. Remarks for the finicky

i) We shall use the expressions "probability of choice" and "market share" equivalently as if one could always, by simple substitution of aggregate for disaggregate data, carry over the interesting properties of each functional forms or model.

ii) There is at present both a tendency to consider models like the LOGIT as a system of consumer demand equations (Tyrell and Mount, 1982) and a tendency to emphasize the distinction between choice models and demand models (Truong, 1981), notably when quantities of trips are analysed instead of expenditure shares. We shall proceed as if the LOGIT and other models of the family to be introduced obviously performed as demand models.

iii) The formal links between utility theory and the most recent members of the family of models presented are not always well-known: this is not surprising for new models formulated on the basis of their reasonable properties and intuitive appeal. We shall proceed under the assumption that, if models are useful, interesting derivations of them will eventually be forthcoming.

iv) All the members of the family to be presented contain, as a nested special case, the linear exponent LOGIT model; they are therefore in the spirit of our use of flexible forms with models of level. However, as little work has been done on the specification of the error terms associated with these (aggregate) models, we shall not emphasize the impact of that specification on the selection of the appropriate market share model.

v) The estimation procedures for members of the family are not completely developed; we will therefore not be able to provide

references to documented computer programs even if preliminary versions of these programs exist.

vi) Although it should not perhaps be said, we have avoided the presence of integrals in the formulation of models, although not always in the plumbing necessary to obtain statistically meaningful results.

B. Starting point: the LIN-LOGIT

The LOGIT mode choice model can be written

$$p(i) = \frac{e^{V_i}}{\sum_{j=1}^{M} e^{V_j}} \tag{11}$$

where

p(i) is the probability of choosing the i^{th} alternative, if one uses disaggregate data, and the market share of that alternative, if one uses aggregate data;

V_i called the utility function of the i^{th} alternative, is a linear function of the characteristics ${}^{i}P_k$ of that alternative (price, etc.) and of the socioeconomic characteristics (Y_s) of those who choose. More precisely, we have

$$V_i = \beta_{io} + \sum_{h=1}^{P} \beta_{ik}[f_{ik}({}^{i}P_k)] + \sum_{s=1}^{S} \beta_{is}[f_{is}(Y_s)] \tag{12}$$

where i=1,...,M and the f(•) functions are identical among alternatives, namely such that, for given k or s and i,j=1,...,M:

$$\text{and} \quad \begin{aligned} f_{ik}(\cdot) &= f_{jk}(\cdot), \text{ for all i and j} \\ f_{is}(\cdot) &= f_{js}(\cdot), \text{ for all i and j but one} \end{aligned} \tag{13}$$

but the f(•) may vary across the P and S variables.

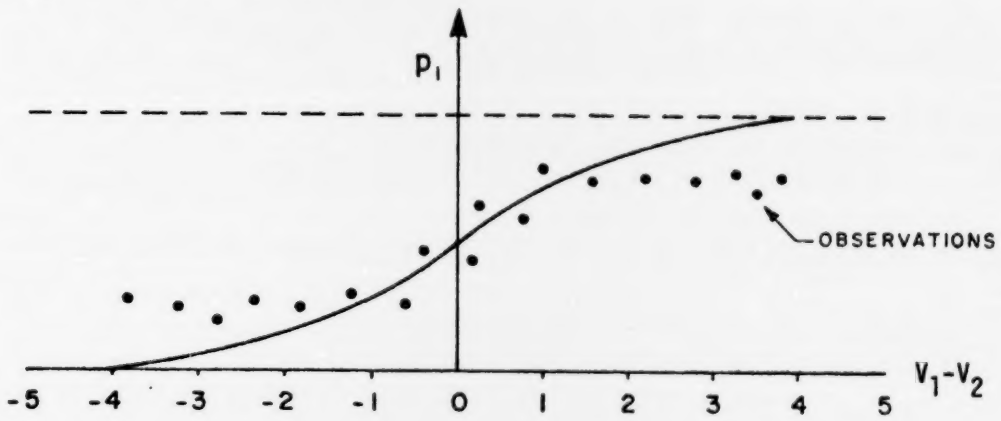
It is well known that this model can be derived from entropy maximization and utility maximization. It is less well known that, as Truong (1981) has shown, it has strong family ties with Theil's (1975) model of random demand, But past and future derivations of the model need not detain us: for our purposes, the *intuitive* interpretation of the V_i so-called utility functions as measures of the attractiveness of the i^{th} alternative will suffice.

The basic properties of the LOGIT model which are of interest to us are

i) equality of *cross-elasticities* of demand: this property means that, if you introduce a new mode or modify an existing mode, its market gain will come at the expense of identical percentage changes of the market shares of the other modes. The model is compatible with Luce's axiom of "independence of irrelevant alternatives" (IIA) because the relative propabilities of choice depend only on the characteristics of these alternatives:

$$\frac{p_i}{p_i} = \frac{e^{V_i}}{e^{V_i}} \quad ; \qquad (14)$$

ii) *symmetry* of the demand curve: the demand curve, shown for a two-mode case in Fig. 3, is symmetrical



RESPONSE CURVE OF BINOMIAL SYMMETRIC LOGIT MODEL

FIGURE 3

with respect to ½. This means that an improved attractiveness of mode 1, relative to mode 2, will progressively increase its choice probability or market share according to a monotone function of the same curvature on both sides of its inflexion point;

iii) *underidentification* of variables identical across alternatives: as is obvious in equation (12), if a variable is common across alternatives, such as income or car ownership, it cancels out of the LOGIT model unless one of the $f_{is}(\cdot)$ differs from the other $f_{js}(\cdot)$. This often leads researchers to associate these variables to only one or a few of the alternatives through a trial-and-error procedure because they follow (13), typically with linear of logarithmic transformations of the variables.

That "linear exponent" or LIN-LOGIT is our starting point.

C. The GENERALIZED LOGIT: the BOX-COX LOGIT and U-LOGIT forms

One approach to the determination of the $f_{ik}(\cdot)$ and $f_{is}(\cdot)$ functions in (12)-(13) is to use Box-Tukey transformations. We investigated (Gaudry and Wills, 1978) this approach with a 4-mode intercity mode choice model containing only, in addition to the mode-specific constants, one fare variable (F) and two service variables, travel time (H) and departure frequency (D). The model is a mode abstract model: $\beta_{ik}=\beta_{jk}$, for all i and j.

We studied a general model, where all three variables have the distinct λ_k's and the same μ_k and the impact of progressively imposing

constraints on that formulation. The results shown in Table 3, indicate how restrictive the LIN-LOGIT, in column [F], can be: it is completely dominated by the log linear formulation, or product market share format, in column [E], which is in turn inferior to the less restricted forms. It is also interesting to note the impact of the BOX-COX form of the conditional t-statistics. Consider Fig. 4 where the three λ_k's are constrained equal and $\mu_k=0$ and compare it to Fig. 5, where the μ_k differs from zero and has the impact of reducing the statistical significance of the modal constants relatively to that of the explanatory variables.

Of course, the t-statistics are biased below and above the point of maximum likelihood ($\lambda_k=-0.2$ in Fig. 4 and $\lambda_k=-0.26$ in Fig.5), but one should also note in both figures that the statistical significance of the modal constants are at a minimum around that point.

This use of transformations suggests that specification error in form might be taken up in these constants intended to capture qualities intrinsic to the mode themselves. This is not altogether surprising: as McFadden said in conversation, "these transformations model how the utility of modal characteristics is perceived by those who choose among the alternatives". Recent work (Wills, 1981b) on another aggregate data set has produced results in which the BOX-COX LOGIT also dominates the LIN-LOGIT.

Despite the incorporation by the Australian Road Research Board, in its BLOGIT computer program for disaggregate data (Crittle and Johnson, 1980), of Box-Tukey and Box-Cox options, applications with disaggregate data are still few. McCarthy (1982) has found only limited gains but Hensher and Johnson (1982) have interesting results.

- The BOX-COX U-LOGIT

In the BOX-COX LOGIT, the Box-Cox transformations are the same, for a given variable, in all utility functions: if we associate appropriate indices to the λ parameters, we have, for any explanatory variable P_k or Y_s,

$$\begin{aligned} \lambda_{ik} &= \lambda_{jk} \qquad \text{for all } i,j=1,\ldots,M, \\ \lambda_{is} &= \lambda_{js} \qquad \text{for all } i,j=1,\ldots,M. \end{aligned} \tag{15}$$

The UNIVERSAL LOGIT (McFadden *et al.*, 1976, Section 5) differs from the LOGIT because the characteristics of all alternatives — their prices, for instance — appear in each utility function. Equation (12) can therefore be rewritten, for i=1,...,M,

$$V_i = \beta_{io} + \sum_{q=1}^{M} \sum_{k=1}^{P} {}^{q}\beta_{ik}[\,{}^{q}f_{ik}({}^{q}P_k)] + \sum_{s=1}^{S} \beta_{is}[\,f_{is}(Y_s)]\,. \tag{16}$$

This model is only used in artificial examples (for instance, Horowitz, 1981) because of the practical difficulty of specifying ${}^{q}f_{ik}(\cdot)$ functions which get around the underidentification problem which arises if identical functions are used for the P variables and all square bracket terms at the numerator and at the denominator of the LOGIT cancel out, as we described earlier for the socioeconomic terms... The use of Box-Cox transformations which differ among themselves solves this problem because the *transformed variables* do not cancel out anymore. Consequently, Box-

TABLE 3

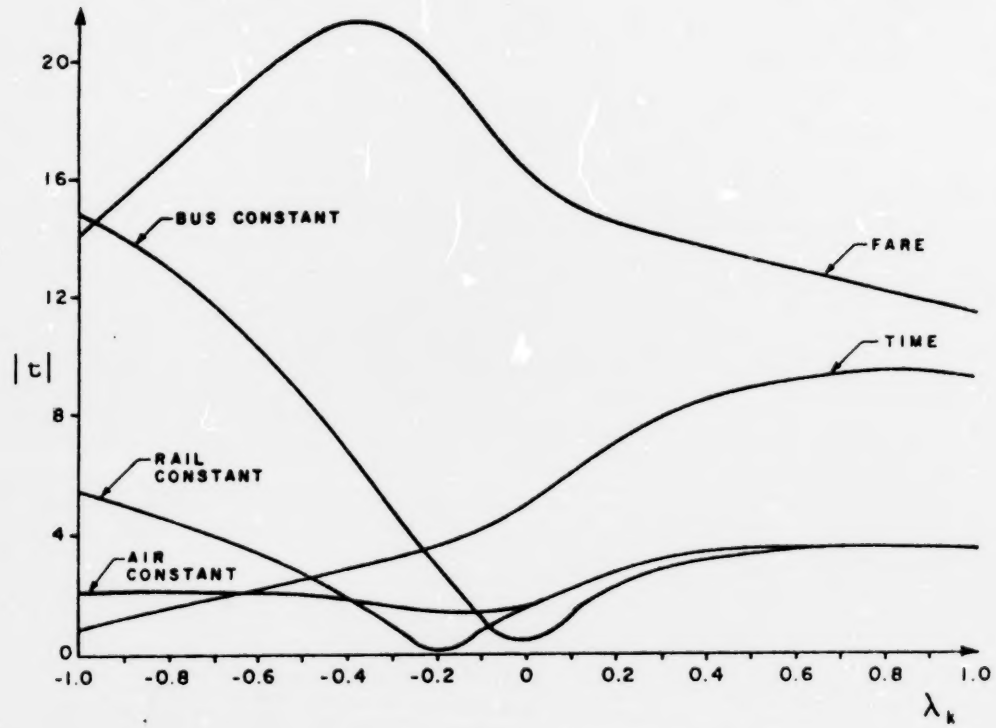
INTERCITY FOUR-MODE MODEL:
BOX-TUKEY, BOX-COX AND LINEAR LOGIT FORMS*

PARAMETERS	MODELS					
	A	B	C	D	E	F
	TLCS-2	TLCS-1	CLCS-2	CLCS-1	Log linear	Logit
(F) $\lambda_{1,1}$	-0.2399	-0.2660	-0.2626	-0.1930	0.0	1.0
(H) $\lambda_{1,2}$	-1.0982	-0.2660	-0.0513	-0.1930	0.0	1.0
(D) $\lambda_{1,3}$	0.0298	-0.2660	0.5712	-0.1930	0.0	1.0
μ_k	35.757	8.6862	0.0	0.0	0.0	0.0
(air) α_{10}	4.7986	0.7288	21.762	0.9343	1.3910	113.00
	(3.629)	(1.044)	(5.001)	(1.462)	(1.695)	(3.486)
(rail) α_{20}	5.0241	0.5087	21.115	0.0380	1.0136	112.28
	(4.230)	(1.124)	(4.957)	(0.097)	(1.618)	(3.459)
(bus) α_{30}	4.2821	-0.3085	20.354	-0.8106	0.2516	111.76
	(3.761)	(-0.793)	(4.837)	(-2.504)	(0.452)	(3.448)
(car) α_{40}	0.0	0.0	0.0	0.0	0.0	0.0
	(-)	(-)	(-)	(-)	(-)	(-)
(F) α_1	-1.8164	-1.7231	-1.8274	-2.2254	-2.9653	-0.841×10^{-3}
	(-14.13)	(-14.96)	(-17.09)	(-18.70)	(-15.57)	(-11.31)
(H) α_2	-0.0153	-0.3932	-0.8358	-0.3605	-1.3148	-0.0141
	(-5.941)	(-5.105)	(-4.596)	(-4.144)	(-5.576)	(-9.092)
(D) α_3	1.3779	0.4414	13.503	0.1331	0.4221	0.0114
	(5.735)	(5.067)	(5.368)	(5.022)	(4.607)	(3.521)
$L_1(\lambda,\mu)$	543.87	539.95	538.66	532.97	528.71	456.32
$L_1(\tilde{\lambda},\tilde{\mu}) - L_1(\bar{\lambda},\bar{\mu})$	0	3.94	5.21	10.90	15.16	87.55
R^2	0.7301	0.7223	0.7197	0.7079	0.6987	0.4909
Skewness	3.155	3.331	3.268	3.466	3.731	6.888
Kurtosis	0.566	0.711	0.624	0.765	0.990	2.667
D.F.	0	2	1	3	4	4

*Numbers in parentheses are t-statistics.

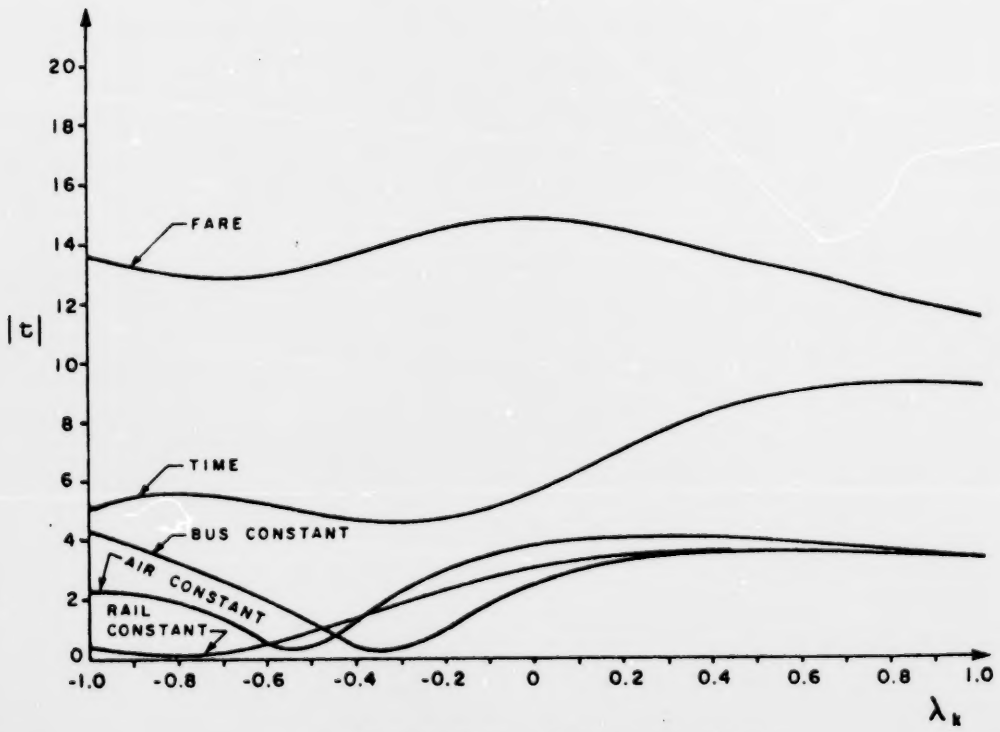
Percentage points of $\frac{1}{2}\chi^2$ distribution with 4 degrees of freedom

Level	0.1	0.05	0.025	0.01	0.005	0.001	0.0001
Value	3.89	4.74	5.57	6.64	7.43	9.23	11.76



INTERCITY FOUR-MODE BOX-COX LOGIT:
ABSOLUTE VALUE OF t-STATISTICS, $\mu_k=0$

FIGURE 4



INTERCITY FOUR-MODE BOX-COX LOGIT:
ABSOLUTE VALUE OF t-STATISTICS, $\mu_k=0$

FIGURE 5

Cox transformations in principle provide a systematic way of including socioeconomic variables in the LOGIT without arbitrarily deciding where identification requirements constrain this or that β_{is} to be zero and simultaneously promise to provide an operational form of the UNIVERSAL LOGIT. Like that model, it does not suffer from the IIA restriction but has a symmetric demand curve.

The BOX-COX U-LOGIT is of interest because we know that the LOGIT is a very bad approximation of reality when reality is generated by the U-LOGIT (Horowitz, 1981): this result is not surprising if one thinks that the absence of the j^{th} price in the i^{th} utility function, responsible for the consistency with the IIA axiom, is analogous to the absence of all prices but one in each equation of a complete demand system; the BOX-COX U-LOGIT, described only in a manuscript (Gaudry, 1978b) is the object of current work,

D. The DOGIT: standard and generalized forms

- The STANDARD DOGIT

The STANDARD DOGIT model can be formulated as

$$p(i) = \underbrace{\frac{1}{(1+\sum_{j=1}^{M}\theta_j)}}_{(A)} \cdot \underbrace{\frac{e^{V_i}}{\sum_{j=1}^{M} e^{V_j}}}_{(B)} + \underbrace{\frac{\theta_i}{1+\sum_{j=1}^{M}\theta_j}}_{(C)} , \qquad (18)$$

or

$$p(i) = \frac{e^{V_i} + \theta_i \sum_{j=1}^{M} e^{V_j}}{(1+\sum_{j=1}^{M}\theta_j)\sum_{j=1}^{M} e^{V_j}} , \qquad (19)$$

where the θ_i are parameters absent from the LOGIT. Among interpretations of this model when all the θ_i are positive, that of a *captivity* model has been derived and proposed by Ben-Akiva (1977). According to this interpretation, expression (C) denotes the probability of being captive, or the captive market share, and (A) the probability of not being captive and of making choices following a LOGIT model (B). In the spirit of Stone's (1954) linear expenditure model, our paper (Gaudry and Dagenais, 1979a) used a *compulsive consumption*, rather than a captivity, interpretation. The former makes it easier to accept negative θ_i, perhaps indicative of "superior" goods in the linear expenditure system (Solari, 1971). The model clearly contains the LOGIT as a nested special case, if all the θ_i equal 0 and generally does *not have constant cross elasticities* of demand but its demand curve is summetric. However, the "captivity levels" introduce variable "thick" tails at each end of the curve instead of the thin tails shown in Fig. 3: the thickness of the tails depend on the value of expression (C) in (18). In contrast with mode specific constants in the LOGIT, which simply make it "harder" to reach the limit of *zero* market share or choice probability, the captivity levels constitute *positive alternative-specific* limits to the probabilities of choice or market shares.

Our first tests of the STANDARD DOGIT with aggregate data (Gaudry and Wills, 1979; Gaudry, 1980) did not show that the model had a decisive advantage over the LOGIT. All comparisons were made with Box-Cox transformations on the explanatory variables in order to provide a convincing comparison of models. Our results might have been much better if we had not imposed the strong nonnegativity constraints on the θ_i to simplify estimation procedures. We are currently working on a relaxation of these constraints.

The STANDARD DOGIT has been the object of Monte-Carlo simulations (Williams and Ortuzar, 1981) in order to compare its performance to that of the LIN-LOGIT. Many persons have shown that the model formalizes the behavior of many analysts of travel mode choice who "pre-assign" some travellers to various modes and distribute the rest according to a LOGIT model (Gunn and Bates, 1980) or shown (Small, 1981) that it is comprises as a special case Westley's (1979) "compressed LOGIT". McFadden (1981) has recently showed that it belongs to the class of "mixed probability random utility" models. A number of researchers are currently applying it with disaggregate data. Although one does not want to anticipate, there are good reasons to thing that the model can be useful.

- the GENERALIZED DOGIT

We have formulated a generalization of the DOGIT in a manuscript (Gaudry and Dagenais, 1978, revised 1981). The model has the following form

$$p(i) = \frac{e^{V_i} + \sum_{j=1}^{M} \theta_{ij} e^{V_j}}{\sum_{h=1}^{M} (e^{V_h} + \sum_{j=1}^{M} \theta_{hj} e^{V_j})} \quad , \quad i=1,\ldots,M \; . \tag{20}$$

It has been the object of preliminary work on a four-mode intercity aggregate data bank and the results (Dagenais *et al.*, 1982) are promising enough to warrant extensive tests which are the object of current investigation. Suffice it to say, at this point, that the presence of the characteristics of all alternatives in the attraction function of each alternative gives it the sort of potential found in the BOX-COX U-LOGIT and in systems of consumer demand equations. Indeed, the matrix of coefficients $\{(\theta_{hj})\}$ can be thought of as a matrix of price coefficients in a system of demand equations.

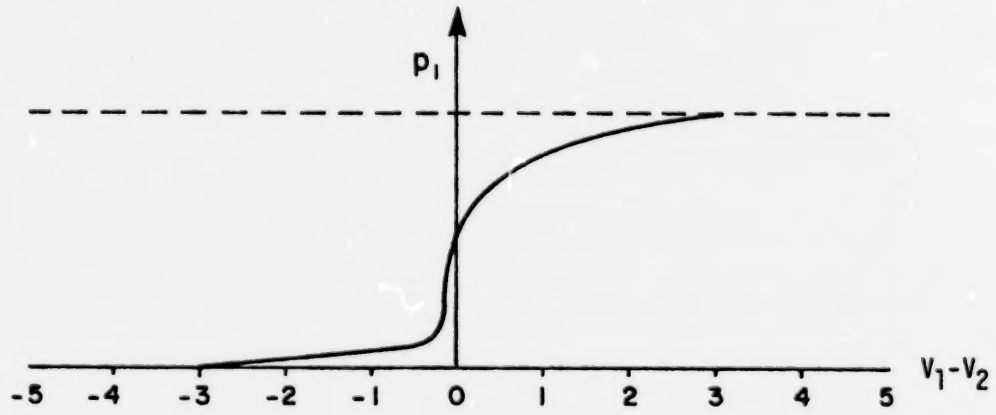
The STANDARD DOGIT is obtained from (20) by adding the following constraints on that matrix :

$$\theta_{i1} = \theta_{i2} = \ldots = \theta_{iM} \; , \quad i=1,\ldots,M \; . \tag{21}$$

The model as presented in (20) is also overparametrised unless the constants contained in the V_i attractiveness functions are implicitly set equal to 0: they are, in practice, absorbed by the θ_{ij} parameters for which unique estimates can be found if the value of one of them is predetermined. In the current work on this model, we allow the θ_{ij} parameters to be negative. Extensive work it still needed to find the conditions in which the GENERALIZED DOGIT, which obviously does not shere the IIA restrictions of the LOGIT, but has many more parameters than the STANDARD DOGIT, is a useful model.

E. The INVERSE POWER TRANSFORMATION model

There is a difficult problem of mode choice theory which is approximately the following: when public transit is improved relative to the car, is it possible to have a very small impact of the price of service level modifications effected until a certain point beyond which there is a large shift to public transit which, upon further modifications, slowly decreases? This is the problem of the presence of *asymmetry* in the reaction function or demand curve of probabilistic choice and market share models. Such asymmetry, shown in Fig. 6, is possible with the INVERSE POWER TRANSFORMATION, or IPT, model.



RESPONSE CURVE OF BINOMIAL ASYMMETRIC IPT MODEL

FIGURE 6

Although the IPT model (Gaudry, 1981) can be applied to both LOGIT and STANDARD DOGIT elements, only the IPT-LOGIT branch has been the object of exploration to date. It can be formulated as

$$p(i) = \frac{\{(\lambda_i e^{V_i}+1)^{1/\lambda_i}+\mu_i\}}{\sum_{j=1}^{M}\{(\lambda_j e^{V_j}+1)^{1/\lambda_j}+\mu_j\}} \qquad (22)$$

where

$$\begin{aligned} \mu_i &> -(\lambda_i e^{V_i}+1)^{1/\lambda_i} \quad , \quad 0 \le \lambda_i \le 1, \\ \mu_i &> |(\lambda_i e^{V_i}+1)^{1/\lambda_i}| \quad . \quad \lambda_j > 1 \ . \end{aligned} \qquad (23)$$

This model, based on the inverse Box-Tukey transformation (7), comprises as nested special cases both the LOGIT (λ_i=1 and μ_i=-1, for all i) and the LOG-WEIBULL ($\lambda_i \to 0$, for all i). Generally speaking, it has three interesting properties.

Firstly, if the λ_i differ among themselves and from 1, the response curve of the model will be asymmetric, as in Fig. 6. One can think of

the λ_i as effectuating a nonlinear transformation of the utility aggregator e^{V_i}. Secondly, the μ_i give variable tails to the response curve of the model, as does the STANDARD DOGIT. For lack of something better, we interpret the μ_i as modeller's ignorance levels which are corrected by the behavioral explanation contained in the V_i functions. Thirdly, as the V_i functions can be those of the LOGIT or of the UNIVERSAL LOGIT (the λ_i prevent variables common to all V_i functions from cancelling out and the model from collapsing), the cross elasticities of demand are not constant. Indeed, the model in principle admits of complementary and substitute alternatives. We have found strong evidence of asymmetry, modeller ignorance, UNIVERSAL LOGIT dominance and weak evidence of complementarity in a two-mode application (Gaudry and Laferrière, 1981). We are currently examining the difficulty of multinomial applications.

4. CONCLUSION

The common denominator of this overview of current work has been the usefulness of general functional forms, which include known forms as nested special cases, for both models of level and probabilistic or market share models. In the first section we have also emphasized the importance of models of residuals which obtain as much systematic information as possible in order to increase the efficiency of the estimates of parameters of the "intelligent" fixed part. In the second section we have presented models even when their close links with utility theory were not yet established. We have left to other hands or other times the detailed discussion of the use of disaggregate data in models of level and in probabilistic choice models and concentrated our presentation on aggregate data applications based both on time-series and on cross-sectional data.

ACKNOWLEDGEMENTS

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